Cryptography - Principles –Cryptographie et Sécurité des Communications–

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Context

Previously

- Caesar Cipher
- One-Time Pads
- Enigma
- **Cryptology = Cryptography + Cryptanalysis**

Today's objectives

- Encryption / Decryption (Confidentiality)
- Verification (Integrity)
- Signature (Authenticity)

Kerchoffs Principle (in “La Cryptographie Militaire” 1883)

1° Le système doit être matériellement, sinon mathématiquement, indéchiffrable ;

2° Il faut qu'il n'exige pas le secret, et qu'il puisse sans inconvénient tomber entre les mains de l'ennemi ;

3° La clef doit pouvoir en être communiquée et retenue sans le secours de notes écrites, et être changée ou modifiée au gré des correspondants ;

4° Il faut qu'il soit applicable à la correspondance télégraphique ;

5° Il faut qu'il soit portatif, et que son maniement ou son fonctionnement n'exige pas le concours de plusieurs personnes ;

6° Enfin, il est nécessaire, vu les circonstances qui en commandent l'application, que le système soit d'un usage facile, ne demandant ni tension d'esprit, ni la connaissance d'une longue série de règles à observer.

- The adversary knows the system [Shannon]
- $\neq$ Security by Obscurity
- Largely accepted in cryptography
- Can be more widely applied to InfoSec (Information System Security) in general.

Confusion and Diffusion (Shannon, 1949)

Confusion

- Each bit in the ciphertext should **depend on several parts of the key**
- Usually implemented using **Substitutions**, aka S-Boxes

Diffusion

- Encryption/decryptions should imply an **avalanche effet.** Precisely (in the original Shannon description): changing a single bit in the plaintext changes half of the bits in the cipher-text (eg at the block granularity)
- Usually implemented using **Permutations** (P-Boxes)

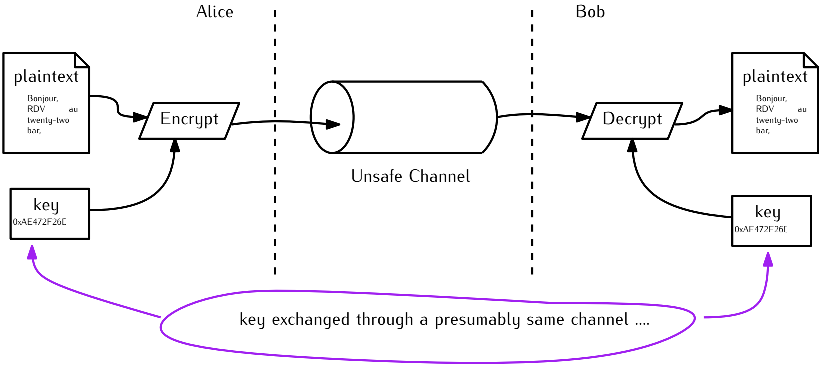
Precautions

- Use recognized libraries (eg OpenSSL), not your own implementation
- Prefer open-source implementations (easier to identify bugs and backdoors)[1]
- In this class, a lot of simplified versions (same on wikipedia)

[1] https://www.theguardian.com/world/2013/sep/05/nsa-how-to-remain-secure-surveillance

Symmetric Cryptography

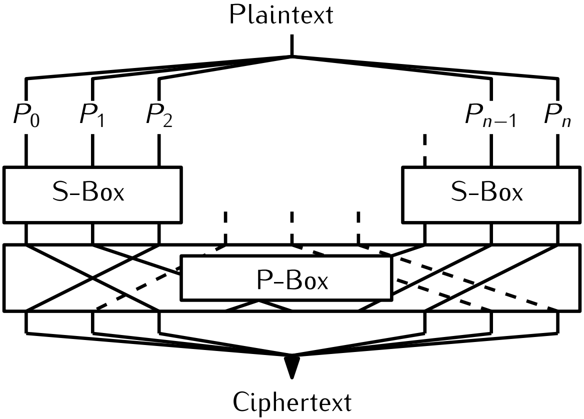
Symmetric Cryptography - Principles



- Encryption, Decryption, Signature and Verification use the same key
- Used implementations are quite efficient.
- A key for each pair of communicating entities

⇒ Rapid explosion in the number of keys

Symmetric ciphers - Basic Principles



Built as a network of substitution/permutation functions:

- Substitution: replace *n* bits by a pre-determined (but moving) table. Must be one-to-one (to allow reversibility of encryption function)
- Permutation: exchange bits

Symmetric ciphers - Basic Principles

Block cipher

- Treat input as fixed-size blocks (between 64 and 128 bits)
- (+) More secure
- (−) Requires padding

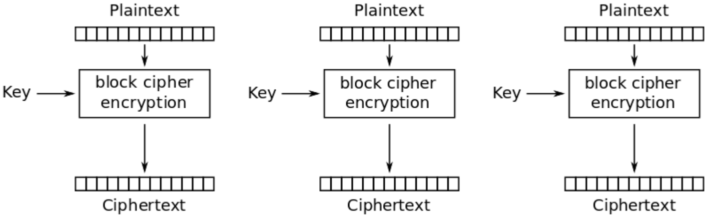
Stream cipher

- Treat input one byte at a time
- The encryption of one byte depends on the current state of the cipher (hence of its history of encryption),
- (+) fast HW implementation
- (−) Security less guaranteed

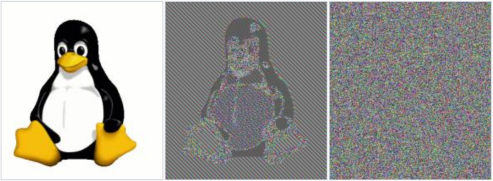
Symmetric ciphers - Operation Modes

Electronic Code Book:

- Message is divided into blocks and each block is encrypted/decrypted separately



- Lacks diffusion

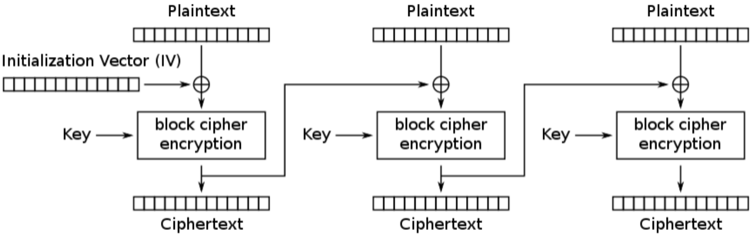


Original image | Encrypted using ECB mode | Modes other than ECB result in pseudo-randomness

Symmetric ciphers - Operation Modes

Cipher Block Chaining

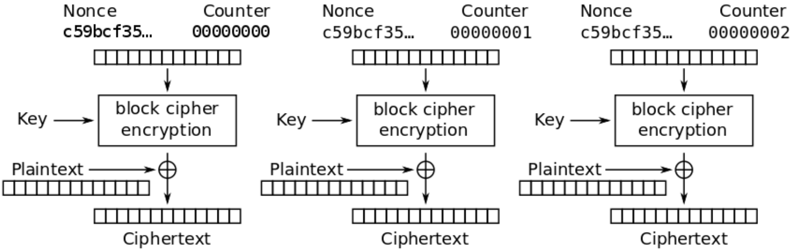
- Initialization Vector to make all cipher message unique



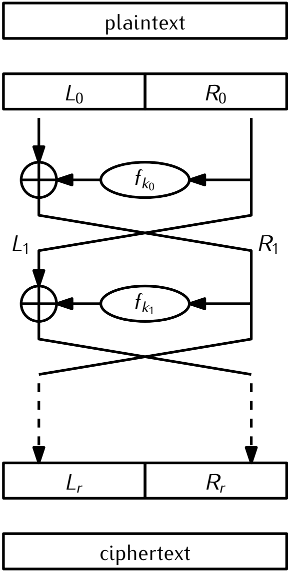
⛔ encryption cannot be parallelized

Symmetric ciphers - Operation Modes

CounTeR



Feistel

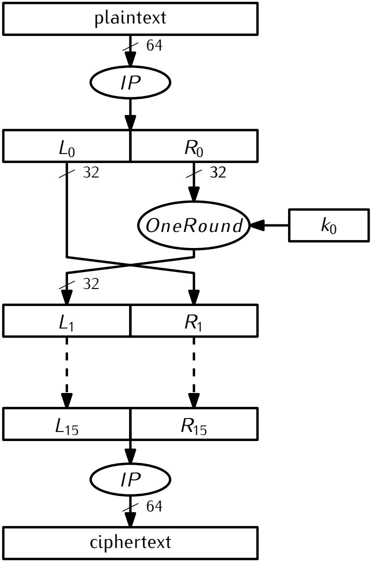


- block cipher
- r rounds
- key k is spilt into r subkeys: $(k_0, \ldots, k_{r-1})$
- plaintext = (L_0, R_0)
- $(L_{i+1}, R_{i+1}) = (R_i, L_i \oplus f_{k_i}(R_i))$
- General structure used in all other ciphers

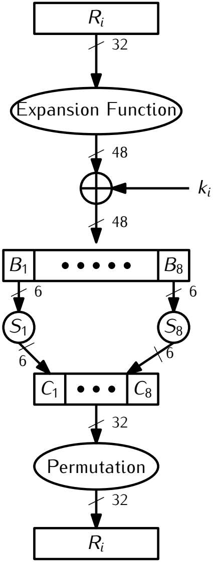
Symmetric Cryptography - DES

- Expands Feistel algorithm, by introducing:
  - More permutations
  - Substitution Boxes (S-Boxes)
- Designed (and initially published) in 1975.
- Block-cipher

DES - General Algorithm



DES - One Round



48-bits subkey obtained through a key-schedule algorithm using the original 64-bits key as input

DES - Weaknesses and Attacks

- Key size in DES was reduced from 128 bits to 56 bits (after discussions with NSA) “to fit on a single chip”
- Practically cracked (brute-forced) in 1997
- Most practical attack to date: still brute force (ie trying out all possible key in turn).
- Attacks faster than brute-force:
  - Differential cryptanalysis: requires 2^{47} chosen plaintexts
  - Linear cryptanalysis: requires 2^{43} chosen plaintexts

Example: Differential Cryptanalysis

Principle:

- Choose two plaintexts x and y s.t.[2]:
  $y = x \oplus \Delta_x$
- Compute the corresponding cyphertexts and for each S-Box S:
  - $S(x, k_i)$
  - $S(y, k_i) = S(x \oplus \Delta_x, k_i)$
- Compute difference on S-Boxes:
  - $\Delta_y = S(x \oplus \Delta_x, k_i) \oplus S(x, k_i)$
- Repeat this for many plaintexts and several key hypothesis $k_i, i \in \{0, n\}$
- key k_j that minimizes Δ is deemed “most probable”.

Limits:

- In practice requires 2^{47} well-chosen plaintext (so that Δ_x is “not too big”)
- Limits: choose the “right” plaintexts

[2] $\oplus$ = “xor”

Bonus: why $\oplus$ (xor) is "difference"?

$\oplus$	0	1
0	0	1
1	1	0

Which means $x \oplus y = 1$ iff $x \neq y$

3DES

- Standardized in 1998 to compensate for the weaknesses of DES
- DES has a 56-bits key
- 3DES **chains 3 DES together**:
  - Encrypt = Encrypt(k_1)$\rightarrow$Decrypt(k_2)$\rightarrow$Encrypt(k_1)
  - Decrypt = Decrypt(k_1)$\rightarrow$Encrypt(k_2)$\rightarrow$Decrypt(k_2)
  - Key: 112 bits ($k_1|k_2$)
- Developped in parallel of AES (waiting for AES to be defined)

AES - Advanced Encryption Standard

- Supersedes DES
- Standardized in 2001
- NIST-organized competition with 5 finalists:
  - IBM proposed MARS
  - RSA proposed RC6
  - Serpent by Anderson, Bihman, Knudsen
  - Twofish by Bruce Schneier et al
  - Rijndael, by Daemen and Rijmen
- Rijndael's was elected by community after a thourough international comparative effort (including NSA, companies, academics), based on security, performance (speed, memory usage).
- NB: no-patent allowed (imposed by the NIST)

AES - Principle[3]

- AES operates on 4×4 array of 16 bytes, called **the state**

$$\begin{bmatrix} b_0 & b_4 & b_8 & b_{12} \\ b_1 & b_5 & b_9 & b_{13} \\ b_2 & b_6 & b_{10} & b_{14} \\ b_3 & b_7 & b_{11} & b_{15} \end{bmatrix}$$

- Key size specifies the number of transformation rounds to convert input plaintext into output ciphertext:
  - 10 rounds for 128-bit keys
  - 12 rounds for 192-bit keys
  - 14 rounds for 256-bit keys

[3] https://en.wikipedia.org/wiki/Advanced_Encryption_Standard

AES - Algorithm (for 10 rounds)

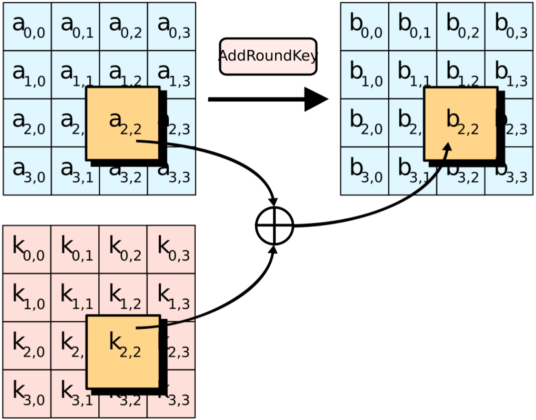
```
void AES_Run_secure(void){
  int i;
  addRoundKey();
  for(i = 0; i < 9; i++){
      subBytes();
      shiftRows();
      mixColumns();
      addRoundKey();
  }
  subBytes();
  shiftRows();
  addRoundKey();
}
```

AES - Initialization

- **KeyExpansion** — round keys are derived from the cipher key using the AES key schedule. AES requires a separate 128-bit round key block for each round plus one more.
- Initial State = Input plaintext

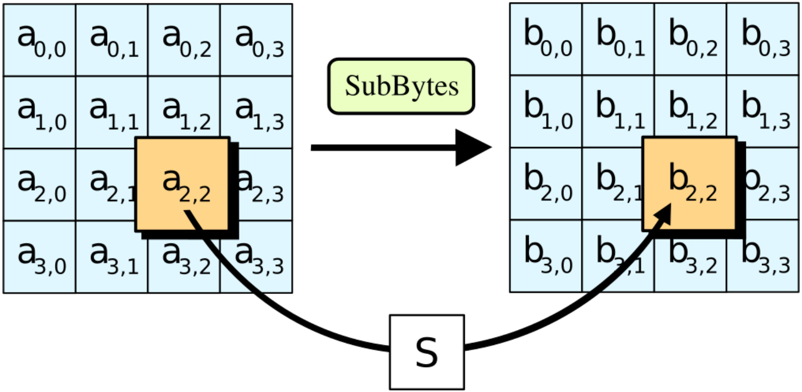
AES - Round Key Addition

- **AddRoundKey** – each byte of the state is combined with a byte of the round key using bitwise xor.



AES - SubBytes

- SubBytes = a non-linear substitution step where each byte is replaced with another according to a lookup table.
- lookup table = S-box



AES - SubBytes

- SubBytes = a non-linear substitution step where each byte is replaced with another according to a lookup table.
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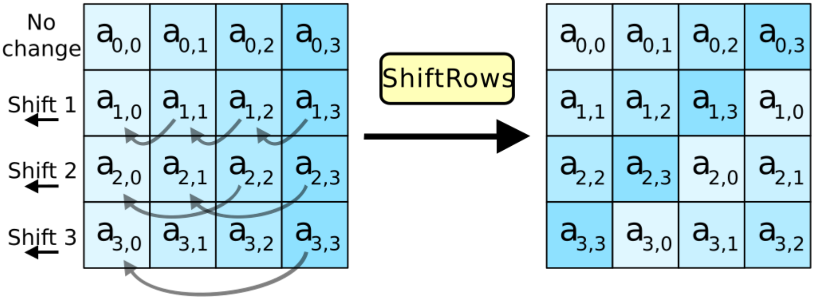
AES S-box

	00	01	02	03	04	05	06	07	08	09	0a	0b	0c	0d	0e	0f
00	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
10	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
20	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
30	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
40	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
50	53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
60	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
70	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
80	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
90	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
a0	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
b0	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
c0	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
d0	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
e0	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
f0	8c	a1	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16

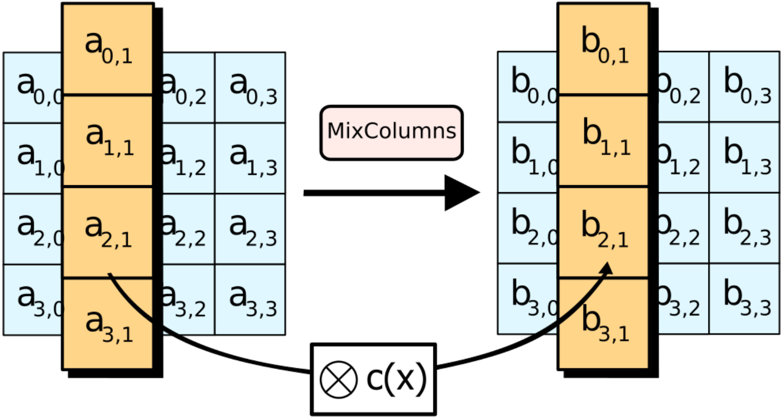
AES - ShiftRows

ShiftRows = a transposition step where the last three rows of the state are shifted cyclically a certain number of steps.



AES - MixColumns

- MixColumns = a linear mixing operation which operates on the columns of the state, combining the four bytes in each column. AddRoundKey
- Together with ShiftRows, MixColumns provides diffusion in the cipher.



AES - One Round

One Round ==

```
subBytes();
shiftRows();
mixColumns();
addRoundKey();
```

- Repeat 9, 11 or 13 rounds
- Plus an extra one without the MixColumns

AES - Weaknesses and Attacks

Related-key attacks exists

- $2^{99.5}$ time and space complexity
- btw: age of universe ˜ 2^{70}
- Anyway totally impractical (because keys are well-chosen to be independant in crypto-systems)

Side-channel attacks are practical

- 6-7 blocks plaintexts needed

$\Rightarrow$ requires HW protections

Symmetric Cryptogaphy - Conclusions

- ➕ Overall very effecient (linear in the size of data to encrypt)
- ➕ Arithmetic/Logical operations are simple: xor.
- ➖ Requires a shared key!
- Solutions to this:
  - Avoid the need for a common key
  - Find a way to securely share a common key

Key Sharing Problem

- Symmetric cryptography uses same key to encrypt and decrypt
- Problem: how to share this key
- Hypothesis: there is no secure channel to exchange the key

Diffie-Hellman Key Exchange

Alice

let's agree on :
p = 23 (prime)
and g = 5
(primitive root modulo 23)

Bob

Alice chooses a secret key P_A

Bob chooses a secret key P_B

g^{P_A} mod p →

← g^{P_B} mod p

Alice computes
$(g^{P_B} \bmod p)^{P_A} \bmod p$
$= g^{P_B P_A} \bmod p$

Bob computes
$(g^{P_A} \bmod p)^{P_B} \bmod p$
$= g^{P_A P_B} \bmod p$
$= g^{P_B P_A} \bmod p$

Alice and Bob now share a common key $g^{P_A P_B}$ mod p

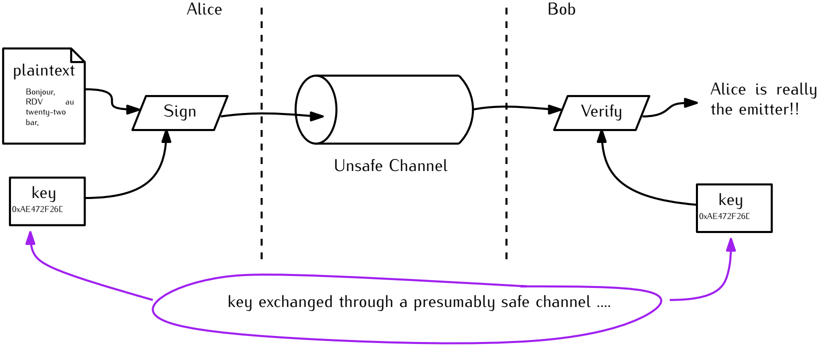
NB1: $g^k mod n$ is the **modular exponentiation** and can be computed quiet efficiently....

NB2: g is a primitive root modulo n if $\forall a$ (integer) coprime to n, $\exists k$ for which $g^k \equiv a (mod n)$.

NB3: The strength of the scheme comes from the fact that $g^{P_A P_B} mod p = g^{P_B P_A} mod p$ take extremely long times to compute by any known algorithm just from the knowledge of p, g, $g^{P_A} mod p$, and $g^{P_B} mod p$.

Hash

Cryptographic Hash



- eg Hash-based Message Authentication Code
- Only sender and recipient can sign/verify the message

Cryptographic Hash - Principle

- Compute a “footprint”
- The message can be of any size, the footprint is of fixed size
- Pseudo-unique identification of message
- Used for:
  - Integrity checks
  - Cryptographic signature
  - PRNG
  - Hashed password storage

Cryptographic Hash - Good Properties

- **Pre-image resistance**: no one can reverse the hash function (to find input from output)
- **Second pre-image resistance**: unicity of hash. Given an input and the corresponding hash, one cannot find another input with the same hash.
- **Collision-resistance**: no-one can produce two different inputs with the same hash
- **Randomness**

Cryptographic Hash - today' state of affairs

Existing (and used) implementations

- MD5: please don't use anymore: "cryptographically broken and unsuitable for further use"
- SHA-1: not recommanded anymore (since 2017)
- SHA-2: still not planned for removal
- SHA-3: standardized in 2015

Current situation

- Hash functions are critical in crypto!
- SHA-2 is still safe but is conceptually close to SHA-1 and might share some weaknesses with it
- SHA-3 considered "as safe" but built completely differently

Asymmetric Cryptography

(general) Asymmetric Cryptography

- Each participant u has a pair of keys $(Pub_u, Priv_u)$.
- u sends Pub_u to v
- v sends Pub_v to u
- u can encrypt its messages to v using a combination of Pub_v and $Priv_u$
- v can decrypt messages from u using a combination of Pub_u and $Priv_v$

Note:

- Relies on “hard mathematical problems”:
  - Discrete logarithm
  - Factorization of large numbers
- Usually slow (exponentiation)

RSA

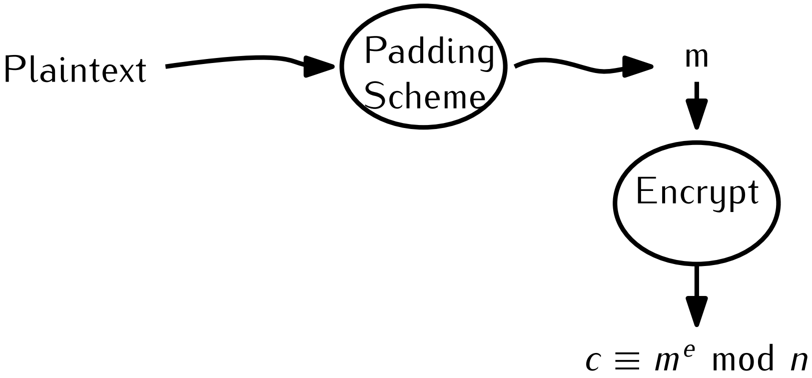
- Invented in 1977 by Rivest, Shamir and Adleman
- MIT Patent in 1983, expired in 2000
- Security based on the difficulty of factorizing large integers

RSA - Key generation

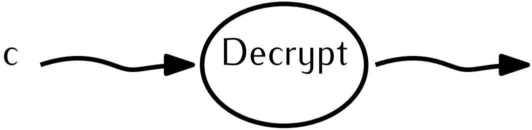
- Choose p and q, two prime numbers: random, kept secret
- Compute $n = pq$
- Compute $\lambda(n)$,
  - $\lambda(n) = lcm(\lambda(p), \lambda(q))$
  - $= lcm(p-1, q-1)$
  - $= \frac{pq}{gcd(p,q)}$ ... (gcd obtained with Euclid algorithm)
- Choose e s.t.:
  - $1 < e < \lambda(n)$
  - $gdc(e, \lambda(n)) = 1$
- Compute $d = e^{-1} \textbf{ mod } \lambda(n)$
  - d is the "private key exponent"

$Pub = (e, n)$ $Priv = (d, n)$

RSA - Encryption



RSA - Decryption



$$m \equiv c^d \mod n$$

RSA - Example

1. $p = 61$ and $q = 53$
2. n = pq = 3233
3. $\lambda(n) = lcm(p - 1, q - 1)$
4. $= \lambda(3233) = lcm(60, 52) = 780$
5. Choose $1 < e < 780$ (coprime to 780), eg $e = 17$
6. $d = e^{-1} \bmod \lambda(n)$
7. $= 413$ (as $1 = 17 * 413 \bmod 780$)
8. Public key = $(e = 17, n = 3233)$
9. Private key = $(d = 413, m = 3233)$
10. $c(m) = m^{17} \bmod 3233$
11. $m(c) = c^{413} mod 3233$
12. $m = 65 \rightarrow c = 65^{17} \bmod 3233 = 2790$
13. $2790 \rightarrow m = 2790^{413} \bmod 3233 = 65$

RSA - Properties & Limitations

- ➕ Finding d requires factorizing n (if finding p and q s.t. $n = p * q$: proven difficult (for p and q large)
- ➖ Implementation is tricky : good PRNG, acceptable e
- ➖ Relies on exponentiation which is **expensive** :

$$x^y = \underbrace{x * x * \ldots * x}_{y \text{ times}}$$

  - Requires a (fast) multiplier
  - y is big (if you want security)
  - Way more expensive than xor !

Key management

The key distribution problem

- To encrypt a message or check a signature, Alice needs Bob's public key
- Otherwise, it may encrypt a message thinking only Bob will read it, but maybe Charlie can read it instead
- How can she get this public key in a secure manner?
- Hard problem, no perfect solution

Note: Using the right key guarantees Bob **is** Bob, but not that Bob is honnest ...

Existing solutions

- Hierarchical certification authorities
- Web of trust (eg PGP)
- Direct exchange of keys

Hybrid Cryptography

Comparing Symmetric / Asymmetric cryptogaphy

Symmetric cryptography

- 1 key per pair of participants (n^2 keys)
- Fast: simple operations, easy to implement in HW

Asymmetric cryptography

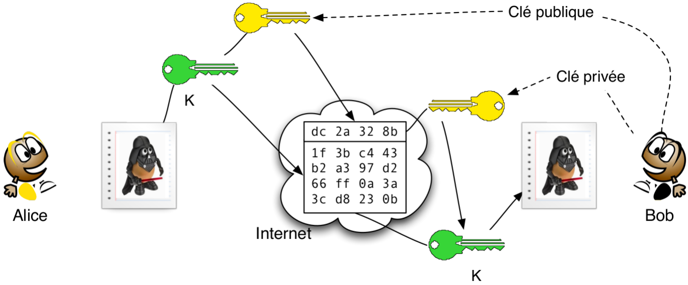
- 1 pair of key per participant ($2n$ keys)
- Slow: complex operations, eg exponentiations

Hybrid cryptography

- Alice encrypts a symmetric key with the public key of Bob
- Alice encrypts the message with the symmetric key

$\Rightarrow$ Best of both worlds

The “best of both worlds”



- Alice encrypts message with Symm key *k*
- Alice encrypts *k* with Bob’s public key
- Bob decrypts *k* with his private key
- Bob decrypts message with *k*

Next time

- Cryptographic protocols
- Public Key Authorities
- PGP